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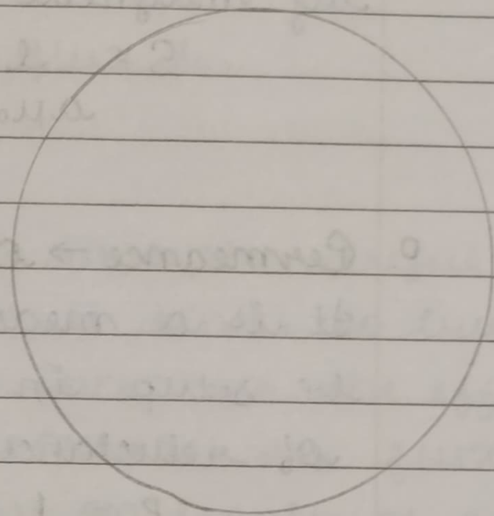
Unit-III
Magnetic circuits

Consider a magnetic ring having high permeability as shown in figure.

The solenoid having n turns & current I will flow through it. Magnetic flux ϕ will setup due to this current in the core.



Let, l = mean length of circuit (in metre)

a = Area of cross-section in m^2.

μ_r = Relative permeability of the material.

So, magnetic flux density of m^{th} core;

$$B = \frac{\phi}{a} = wb/m^2$$

& Magnetic force;

$$H = \frac{B}{\mu_0 \mu_r} = \frac{\phi}{a\mu_0\mu_r} = AT/m$$

According to work Law, the work done in moving a unit pole once round the magnetic ckt is equal to Ampere turns enclosed by the magnetic circuit.

$$H \cdot l = NI$$

$$\Rightarrow \frac{\phi}{a\mu_0\mu_r} \times l = NI$$

$$\Rightarrow \phi = \frac{NI a \mu_0 \mu_r}{l} = \frac{NI}{(l / a\mu_0\mu_r)}$$

$$\Rightarrow \phi = \frac{NI}{S} = \frac{MMF}{S} \quad \left\{ \text{Reluctance} = S = \frac{l}{a\mu_0\mu_r} \right\}$$

- **Reluctance:**
  The opposition offered to the magnetic flux by magnetic circuit is called reluctance.

$$S = \frac{l}{a\mu_0\mu_r} = \frac{\text{Ampere Turns}}{\text{weber}} = AT/Wb$$

- **Permeance → Conductance:**
  It is a measure of ease with which flux can be setup in a material. It is just reciprocal of reluctance of material.

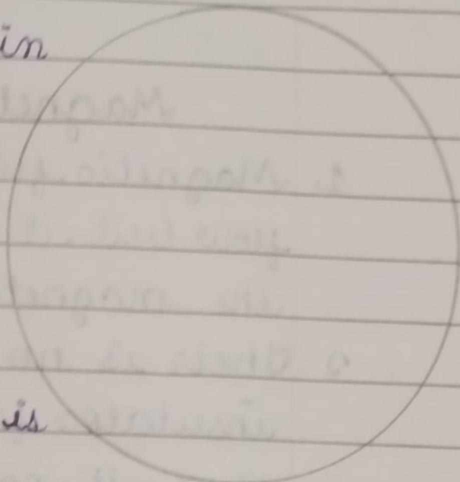
$$P = \frac{1}{S} = \frac{a\mu_0\mu_r}{l} = Wb/AT \text{ or Henry}$$

- **Specific Reluctance/ → Resistivity :** (Reluctivity)
  It is specific reluctance of the material.

> **Leakage Flux & Fringing:**
The magnetic flux which does not follow the intended (pre-decided) path in a magnetic circuit is called leakage flux.

When a current is passed through a solenoid magnetic flux is produced by it which is shown in figure, below.

Most of this flux is setup in a magnetic core & passes through the air gap. This flux is known as useful flux ϕ_u.



However, some of the flux is just setup around the coil & is not utilized for any work.
This flux is called leakage flux ϕ_L.
So, $\phi_T = \phi_u + \phi_L$

It is clear from the figure that the useful flux when setup in the air gap. It tends to bulge outwards at B & B′. This increases the effective area in air gap & decreases the flux density. Fringing depends on length of air gap.

Comparision b/w Magnetic & Electric Circuits:-

Similarities

Magnetic circuit	Electric Circuit
1. Closed path for magnetic flux is called magnetic ckt.	1. The closed path for electric current is called electric circuit.
2 i Flux	2. i Current
ii MMF	ii. EMF
iii Reluctance	iii. Resistance
iv. Permeance	iv ~~Current Density~~ Conductance
v. Permeability	v Conductivity
vi. Reluctivity	vi. Resistivity
vii. Flux Density	vii. Current Density
viii Magnetic Intensity	viii Electric Intensity

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Dissimilarities

Magnetic Circuit	Electric Circuit
1. Magnetic flux does not flow but it sets up in the magnetic circuit.	1. The electric current actually flows in an electric circuit.
2. Their is no perfect insulator for magnetic flux, it can be setup even in non-magnetic materials like air, rubber glass, etc. with reasonable MMF.	2. For electric current their are large number of perfect insulators like air, rubber, glass, etc which do not allow current to pass through them.
3. The reluctance of magnetic circuit is not constant rather it varies with the value of B. It is because the value of μ_r changes considerably with change in B.	3. The resistance of electric circuit is almost constant. As its value depends upon the value of ρ (resistivity), which is almost constant. However the value of ρ & R may vary slightly if temp. changes.
4. Once the flux is setup in a magnetic circuit no energy is expanded However a small amount of energy is required at the start to create flux in the circuit.	4. Energy is expanded continously as long as the current flows through the circuit. This energy is dissipated in the form of heat.

S: Series Magnetic Circuit:-
consider a magnetic ring as shown in figure with 3 different magnetic materials & an air gap. Now,

Total MMF = $\phi \times S$

Total Reluctance (S) = $S_1 + S_2 + S_3 + S_g$

$$= \left[\frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0}\right]$$

$$\text{Total MMF} = \phi \left[\frac{l_1}{a_1 \mu_0 \mu_{r1}} + \frac{l_2}{a_2 \mu_0 \mu_{r2}} + \frac{l_3}{a_3 \mu_0 \mu_{r3}} + \frac{l_g}{a_g \mu_0}\right]$$

$$\text{Total MMF} = \left[\frac{\phi l_1}{a_1 \mu_0 \mu_{r1}} + \frac{\phi l_2}{a_2 \mu_0 \mu_{r2}} + \frac{\phi l_3}{a_3 \mu_0 \mu_{r3}} + \frac{\phi l_g}{a_g \mu_0}\right]$$

We know that, $B = \phi / a$

$$\text{Total MMF} = \frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0}$$

$$\because H = \frac{B}{\mu_0 \mu_r}$$

$\therefore$ Total MMF = $H_1 l_1 + H_2 l_2 + H_3 l_3 = \phi S = NI$

Total MMF = Total Ampere Turns = $\phi \cdot S$

Numericals

1. An iron ring of 400 cm mean circumference is made from round iron of cross-section 20 cm², $\mu_r = 500$. If it is wound with 400 turns, what current would be required to produce a flux of 0.001 wb.

Soln. Given :- $l = 400\text{cm} = 4\text{ m}$, $a = 20 \times 10^{-4}\text{ m}^2$, $\mu_r = 500$
$N = 400$, $\phi = 0.001\text{ wb}$

To find : $I = ?$

Solve :- We know that $\phi S = NI$ $\left\{S = \frac{l}{a \mu_0 \mu_r}\right\}$

$$\Rightarrow I = \frac{0.001 \times 4}{20 \times 10^{-4} \times 4\pi \times 10^{-7} \times 500} \times \frac{1}{400}$$

$\Rightarrow I = 10^{-3+4+7.5} \times \frac{1}{2 \times 4\pi \times 5}$

$\Rightarrow I = \frac{100^{25}}{4 \times 3.14} = 7.96$ Amp

Hence, current of 7.96 Amp would be require to produce a flux of

2. A flux density of $1.2 \, Wb/m^2$ is required in the 2 mm air gap, of an electromagnet having an iron path 1 m long. Calculate the magnetising force & required current, if electromagnet has 1,273 turns. Assume μ_r of Fe is 1500

Soln Given:- $B = 1.2 \, Wb/m^2$, $l_g = 2 \times 10^{-3} \, m$,
$l_{Fe} = 1 \, m$, $N = 1.273$, $\mu_r = 1500$

To find:- $H = ?$, $I = ?$

Solve:- $H_T = H_{Fe} + H_g$

$H_T = \frac{B}{\mu_0}\left(\frac{1}{\mu_r} + 1\right) = \frac{1.2}{4\pi \times 10^{-7}}\left(\frac{1}{1500} + 1\right)$

$H_T = \frac{0.3}{3.14 \times 10^{-7}} \times \frac{1501}{1500} = \frac{450.3 \times 10^7}{4710}$

$H_T = 0.956 \times 10^7$ AT/m

Total AT's $= NI$

$I = \frac{\text{Total AT's}}{N}$

Total AT's $= H_i l_i * H_g l_g$

$= \frac{1.2}{4\pi \times 10^{-7} \times 1500} \times 1 + \frac{1.2}{4\pi \times 10^{-7}} \times 2 \times 10^{-3}$

Total AT's $= 636.95 + 1910.8 = 2547.75$

1 Wb = 10^8 lines of force

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Hence, $I = \frac{2547.75}{1.273}$

$I \simeq 2$ Ampere

Therefore, $H = 955536.6$ AT/m &

$I = 2$ Amp.

3. Estimate the number of Ampere turns necessary to produce a flux of 1 lakh lines round an iron ring of $6\,cm^2$ cross section & 20 cm mean diameter, having an air gap of 2 mm wide across it. Permeability of iron is 1200.

Soln. Given:- $\phi = 10^{5-8}$, $a = 6\times10^{-4}\,m^2$, $Dm = 0.2\,m$

$lm = \pi D = 0.6283\,m$, $lg = 2\times10^{-3}\,m$

$\mu_{ri} = 1200$

To find: Total AT's

Solve:- $li = lm - lg = 0.6263\,m$

Total AT's $= \phi \times S$

$S = Si + Sg$

$= \left(\frac{li}{ai\mu_0\mu_{ri}} + \frac{lg}{ag\mu_0}\right)$

$S = \left(\frac{0.6263}{6\times10^{-4}\times4\pi\times10^{-7}\times1200} + \frac{0.002}{4\pi\times10^{-7}\times6\times10^{-4}}\right)$

$S = \frac{1}{6\times10^{-4}\times4\pi\times10^{-7}}\left(\frac{0.6263}{1200} + \frac{0.002}{1}\right)$

$S = 1.327\times10^9 \quad (0.000522 + 0.002)$

$S = 1.327\times10^9\,(0.002522)$

$S = 334.6694\times10^4$

~~S = 3344.79 AT's~~

Total AT's $= 10^{-3}\times3344.79\times10^3 = 3344.79$

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4. Calculate the relative permeability of iron ring when the exciting current taken by the 600 turn coil is 1.2 Amp. The total flux produced is 1 mWb & mean circumference of the ring is 0.5 m & cross-section is 10 cm².

Solⁿ. Given:- $I = 1.2$ Amp, $N = 600$, $\phi = 1 \times 10^{-3}$ Wb
$a = 10 \times 10^{-4}\ m^2$, $l = 0.5\ m$

To find: $\mu_r = ?$

Solve:- $\phi S = NI$

$$\phi \frac{l}{a\mu_0\mu_r} = NI$$

$$\Rightarrow \mu_r = \frac{\phi l}{a \mu_0 N I}$$

$$\Rightarrow \mu_r = \frac{10^{-3} \times 0.5}{10^{-3} \times 4\pi \times 10^{-7} \times 600 \times 1.2}$$

$$\Rightarrow \mu_r = \frac{500000}{904.32}$$

$\Rightarrow \mu_r = 552.91$

$\therefore \quad \mu_r \approx 552.6$

5. An iron ring of mean length 1 m has an air gap of 1 mm & a winding of 200 turns. If μ_r of iron is 500, when current of 1 Amp flows through the coil. Find flux density.

Solⁿ. Given:- $l_m = 1\ m$, $l_g = 0.001\ m$, $N = 200$, $\mu_{ri} = 500$
$I = 1$ Amp.

To find:- $B = ?$

Solve:- $l_i = 1 - 0.001\ m = 0.999\ m$

$NI = \phi S$

$$\Rightarrow NI = \frac{\phi}{a}\left[\frac{l_i}{\mu_0\mu_{ri}} + \frac{l_g}{\mu_0}\right]$$

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$$\Rightarrow B = \frac{NI}{\left[\frac{l_i}{a\mu_0\mu_{ri}} + \frac{l_g}{\mu_0}\right]} = \frac{200 \times 1 \times 4\pi \times 10^{-7}}{\left[\frac{0.999}{500} + 0.001\right]}$$

$$\Rightarrow B = \frac{2512 \times 10^{-7}}{0.002998}$$

$\Rightarrow B = 0.0838\ wb/m^2$

Hence flux density is $0.0838\ wb/m^2$.

6. A ring has a diameter of 21cm & a cross-section of $10\,cm^2$. The ring is made up of semicircular sections of cast iron & cast steel with each joint having a reluctance equal to an air gap of 0.2mm. Find AT's required to produce a flux of 8×10^{-4} wb. μ_r cast steel = 800 & μ_r cast iron = 166

Solⁿ- Given:- $d = 21cm = 0.21\,m$, $a = 10 \times 10^{-4}\,m^2$.

$l_g = 0.2 \times 10^{-3}\,m$, $\phi = 8 \times 10^{-4}\,wb$

$(\mu_r)_{CS} = 800$, $(\mu_r)_{CI} = 166$

To find: Total AT's

Solve:- Total AT's = $H_g l_g + H_{cs} l_{cs} + H_{CI} l_{CI}$

$$H_g = \frac{B}{\mu_0} = \frac{\phi \times 1}{a\mu_0} = \frac{8 \times 10^{-4}}{10^{-3} \times 4\pi \times 10^{-7}} = \frac{2 \times 10^{-4+3+7}}{\pi} = 636.95 \times 10^3$$

$$H_{cs} = \frac{B}{\mu_0\mu_r} = \frac{\phi}{a\mu_0\mu_r} = \frac{8 \times 10^{-4} \times 2}{\pi \times 0.21 \times 4\pi \times 10^{-7} \times 800} = \frac{1000}{4.15}$$

$H_{cs} = 2.4095$

$$H_{CI} = \frac{\phi}{a\mu_0\mu_r} = \frac{8 \times 10^{-4} \times 2}{10^{-3} \times \pi \times 0.21 \times 4\pi \times 166} = \frac{8000}{687.412}$$

$H_{CI} = 11.64$

Total AT's = $636.95 \times 10^3 \times 0.2 \times 10^{-3} + 2.4095 \times 0.33 + 11.64 \times 0.33$

$= 127.39 + 0.795 + 3.8412$

Total AT's = 132.0262

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7. A mild steel ring of 30cm circumference has a cross sectional area of 6 cm^2 and has a winding of 500 turns on it. The ring is cut through at a point so as to provide an air gap of 1 mm in the circuit. It is found that a current of 4 Amp. in the winding produces a flux density of 1T. Find μ_r of mild steel.

Solⁿ. Given:- $l_{MS} = 0.3\,m$, $a = 6 \times 10^{-4}\,cm^2$, $N = 500$

$lg = 1 \times 10^{-3}\,m$, $I = 4$ Amp, $B = 1T$

To find:- $(\mu_r)_{MS} = ?$

Solve:- Total MMF $= NI = \phi S$

$$NI = \phi [S_i + S_g]$$

$$NI = \frac{\phi}{a}\left[\frac{l_i}{\mu_0 \mu_{ri}} + \frac{l_g}{\mu_0}\right]$$

$$\Rightarrow 500 \times 4 = \frac{1}{4\pi \times 10^{-7}}\left[\frac{0.299}{\mu_{ri}} + 0.001\right]$$

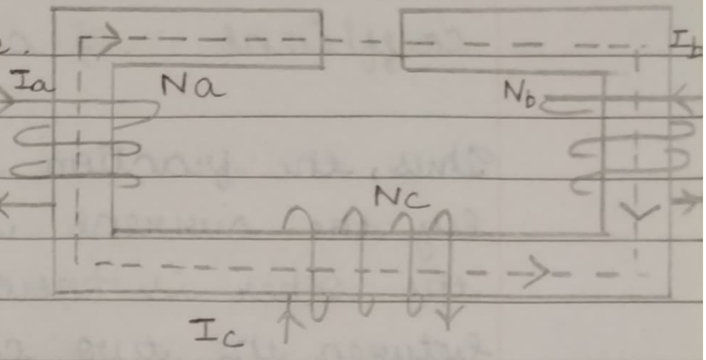
$$\Rightarrow 4\pi \times 10^{-7} \times 2000 = \frac{0.299}{\mu_{ri}} + 0.001$$

$$\Rightarrow 0.0025120 - 0.001 = \frac{0.299}{\mu_{ri}}$$

$$\Rightarrow \mu_{ri} = \frac{0.299}{0.001512} = 197.75$$

8. A rectangular iron core is shown in figure. It has mean length of magnetic path of 100cm, cross-section of 2cm×2cm, permeability of 1400 & an air gap of 5 mm cut in the core. The three coils carried by the core have number of turns $N_A = 335$, $N_B = 600$, $N_C = 600$ & respective currents are 1.6, 4 & 3A respectively. The direction of currents are shown in the diagram. Calculate

flux in the air gap.



Solⁿ- Given:- $l_m = 1m$,
$a = 4 \times 10^{-4} m^2$,
$\mu_r = 1400$,
$l_g = 5 \times 10^{-3} m$, $N_A = 335$,
$N_B = 600$, $N_C = 600$,
$I_A = 1.6 A$, $I_B = 4 A$, $I_C = 3 A$

To find:- $\phi g = ?$

Solve:- Total MMF $= \phi S = NI$

$$\phi [S_i + S_g] = N_a i_a + N_b i_b + (-N_c i_c)$$

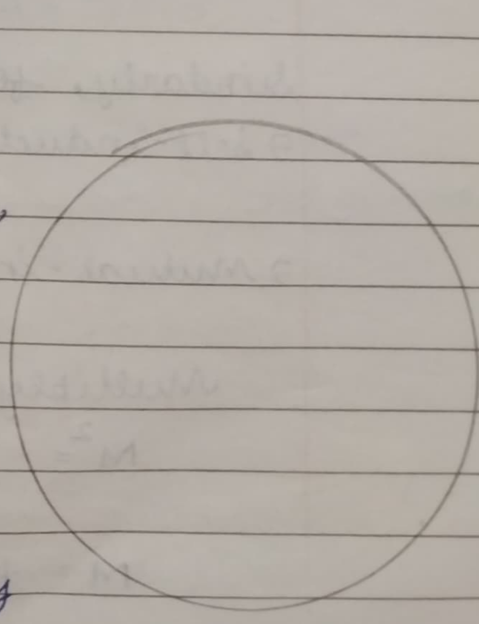
$$\phi = \frac{335 \times 1.6 + 600 \times 4 + 600 \times 3}{\frac{1}{4\pi \times 10^{-7}} \left[\frac{0.995}{4 \times 10^{-4} \times 1400} + \frac{5 \times 10^{-3}}{4 \times 10^{-4}} \right]}$$

$$\phi = \frac{(536 + 2400 - 1800) \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}{[0.000711 + ~~12.~~ 0.0005]}$$

$$\phi = \frac{5.707264 \times 10^{-7}}{~~12.500711~~ \; 0.005711}$$

$\phi = 100 \, \mu Wb$

⇒ Coefficient of Coupling :-



When current flows through coil 1, it produces flux ϕ_1. The whole of this flux may not be linking with other coil couped to it as shown in figure. It may be reduced because of leakage flux ϕ_1 by a fraction of k, known as

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coefficient of coupling.

Thus, the fraction of magnetic flux produced by the current in one coil that links with the other is known as coefficient of coupling, k between the two coils.

If the flux produced by one coil completely links with the other, then the value of k is 1 & coil is said to be magnetically tightly coupled. Whereas if the flux produced by one coil doesnot link at all with the other, then the value of k is ~~known~~ zero & the coil are said to be magnetically isolated.

o Expression for coefficient of coupling:

Considering coil 1;

⇒ Self-inductance of coil 1 = $L_1 = \frac{N_1 \phi_1}{I_1}$ &

⇒ Mutual Inductance of coil 1 = $M = \frac{N_2 \phi_{12}}{I_1} = \frac{N_2 k \phi_1}{I_1}$ — (1)

Similarly, for coil 2;

⇒ Self-inductance of coil 2 = $L_2 = \frac{N_2 \phi_2}{I_2}$ &

⇒ Mutual-inductance of coil 2 = $M = \frac{N_1 \phi_{21}}{I_2} = \frac{N_1 k \phi_2}{I_2}$ — (2)

Multiply eq^n (1) & (2), we get

$$M^2 = \frac{N_1 N_2 \cdot k^2 \phi_1 \phi_2}{I_1 I_2}$$

$$M = k\sqrt{\frac{N_1 \phi_1}{I_1} \cdot \frac{N_2 \phi_2}{I_2}} = k\sqrt{L_1 \cdot L_2}$$

$\Rightarrow$ $M = k\sqrt{L_1L_2}$

$$k = \frac{M}{\sqrt{L_1L_2}}$$

Formulae

Self Inductance	Mutual Inductance
1. $L = \frac{e}{di/dt} = \frac{emf}{di/dt}$	1. $M = \frac{e_M}{dI_1/dt}$
2. $L = \frac{n\phi}{I}$	2. $M = \frac{N_2\phi_{12}}{I_1}$
3. $L = \frac{N^2}{S\text{(Reluctance)}}$	3. $M = \frac{N_1N_2}{S\text{(Reluctance of coil)}}$

Numericals

1. An air cored solenoid has 300 turns. Its length is 25cm & cross-section is 3 cm^2. Calculate self inductance of coil in Henry.

Soln. Given:- $N = 300$, $l = 0.25\ m$, $a = 3\times10^{-4}$

To find:- L (Henry) = ?

Solve:- $L = \frac{N^2}{S} = \frac{N^2 a \mu_0 \mu_r}{l}$ {$\mu_r = 1$ for air cored solenoid}

$$L = \frac{300\times300\times3\times10^{-4}\times4\pi\times10^{-7}}{0.25}$$

$$L = \frac{36\times3.14\times10^{-7}\times3}{0.25}$$

$$L = 0.137\ mH$$

Hence, the self inductance of coil is 0.137 mH.

2. Calculate the inductance of a torroid 25cm mean diameter & 6.25 cm^2 circular cross-section, wound uniformly with 1000 turns of wire. Hence calculate the emf induced when current in it decreases at the rate of 100 Amp/sec.

Solⁿ. Given:- $d_m = 20+5 = 0.25\,m$, $a = 6.25 \times 10^{-4}\,m^2$

$N = 1000$, $di/dt = 100$ Amp/sec

To find:- L & emf = ?

Solve:- $l = \pi D = \pi \times 0.25\,m$.

$L = \frac{N^2}{S} = \frac{N^2 a \mu_0 \mu_r}{l}$ {$\mu_r = 1$ for torroid

$L = \frac{1000 \times 1000 \times 6.25 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1}{\pi \times 0.25}$ (25)

$L = 2500 \times 4 \times 10^{-7}$

$L = 10^{-3}$ Henry

$L = 1\,mH$

& we know, $L = \frac{emf}{di/dt}$

So, $emf = L \times \frac{di}{dt} = 10^{-3} \times 10^2 = 0.1\,V$

Hence, $L = 1\,mH$ & $emf = 0.1\,V$.

3. Two coils A & B of 600 & 1000 turns respectively are connected in series on the same magnetic circuit of reluctance 2×10^6 AT's/wb. Assuming that their is no flux leakage. Calculate

i. Self inductance of each coil.

ii. Mutual inductance between the two coils.

Solⁿ. Given:- $N_A = 600$, $N_B = 1000$, $S = 2 \times 10^6$

To find:- L_A, L_B & M. = ?

Solve:- $L_A = \frac{N_A^2}{S} = \frac{600 \times 600}{20 \times 10^6} = 0.18$ Henry

$L_B = \frac{N_B^2}{S} = \frac{1000 \times 1000}{2 \times 10^6} = 0.5$ Henry

$M = \frac{N_A N_B}{S} = \frac{600 \times 1000}{20 \times 10^6} = 0.3$ Henry

Hence, $L_A = 0.18$ H, $L_B = 0.5$ H & $M = 0.3$ H

- What would be the value of M if $k = 75\%$
- $k = 0.75$ $\left(75\% = \frac{75}{100} = 0.75\right)$

$M = k\sqrt{L_A \cdot L_B}$

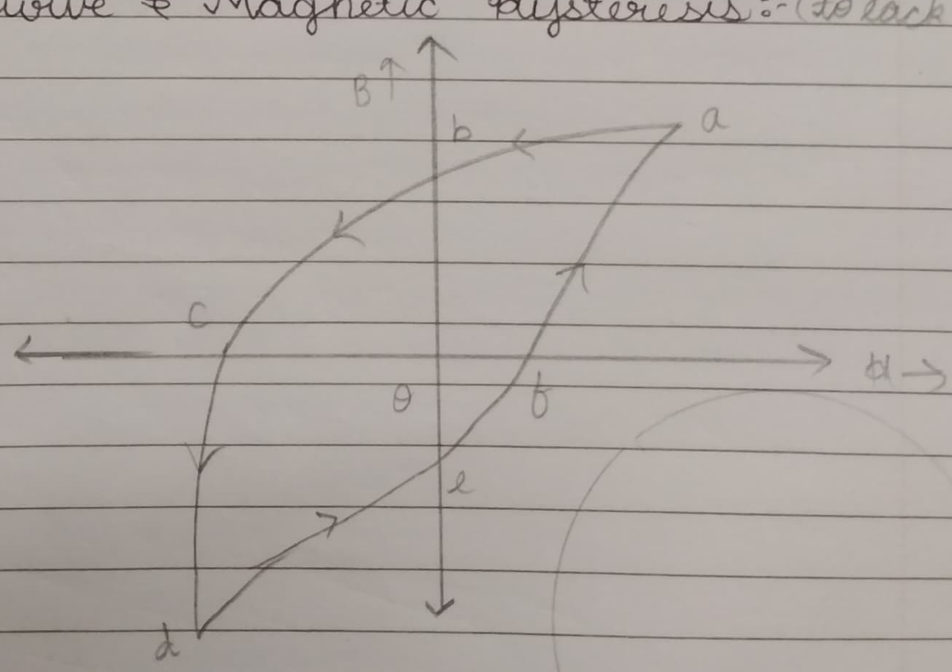
$M = 0.75\sqrt{\frac{18}{100} \times \frac{1}{2}} = 0.75 \times \frac{3}{10}$

$M = \frac{2.25}{10} = 0.225$ H

Hence, M will be 0.225 H if k is 75%.

B-H Curve & Magnetic Hysteresis:- (to back behind)

Retentivity → OB
Residual Magnetism
OC → Coercive force



Lagging behind B w.r.t. H.

Narrow OC wala
materialaccha.